

Soft-Gluon-Pole Contribution in Single Transverse-Spin Asymmetries of Drell-Yan Processes

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Abstract

We use multi-parton states to examine the leading order collinear factorization of single transverse-spin asymmetries in Drell-Yan processes. Twist-3 operators are involved in the factorization. We find that the so-called soft-gluon-pole contribution in the factorization must exist in order to make the factorization correct. This contribution comes from the corresponding cross-section at one-loop, while the hard-pole contribution in the factorization comes from the cross-section at tree-level. Although the two contributions come from results at different orders, their perturbative coefficient functions in the factorization are at the same order. This is in contrast to factorizations only involving twist-2 operators. The soft-gluon-pole contribution found in this work is in agreement with that derived in a different way. For the hard-pole contributions we find an extra contribution from an extra parton process contributing to the asymmetries. We also solve a part of discrepancy in evolutions of the twist-3 operator. The method presented here for analyzing the factorization can be generalized to other processes and can be easily used for studying factorizations at higher orders, because the involved calculations are of standard scattering amplitudes.

1. Introduction

Single transverse-spin asymmetries(SSA) have been observed in various experiments, where an involved hadron is transversely polarized. A review about the phenomenologies of SSA can be found in [1]. In general SSA can be generated if scattering amplitudes have nonzero absorptive parts and there are helicity-flip interactions. It has been shown in production of heavy quarks like top quarks there are sizeable SSA[2, 3]. Because a top quark is heavy, the bound state effects can be neglected. The helicity-flip is due to the nonzero quark mass. Therefore, in the studies of [2, 3] one essentially deals with point-like particles and can use the perturbative theory in standard way.

For SSA involving light hadrons, the origin of SSA is unclear because of bound-state effects and the helicity-conservation of QCD with light quarks which can approximately be taken as massless. However, certain predictions can be made for SSA in cases where large momentum transfers are involved. In these cases, one can use the concept of factorization in QCD. SSA can be factorized in the form of a convolution with nonperturbative matrix elements of hadrons and perturbative coefficient functions. With this form of predictions one is then able to explore hadron structures with experiment. The coefficient functions can be calculated as a perturbative expansion. In this work we study the collinear factorization of SSA in Drell-Yan processes. We will show that the factorization of collinear divergences corresponding to nonperturbative effects is not made in an usual way as factorizations at leading twists.

The collinear factorization for describing SSA has been proposed in [4, 5]. With the collinear factorizations SSA in various processes has been studied in [6, 7, 8, 9, 10, 11, 12]. In such a factorization, the nonperturbative effects of the transversely polarized hadron are factorized into twist-3 matrix elements, or called ETQS matrix elements. Taking SSA in Drell-Yan processes as an example, SSA is factorized as a convolution of three parts: The first part is the standard parton distribution function of the unpolarized hadron defined with twist-2 operators. The second part consists of matrix elements of the polarized hadron defined with twist-3 operators. The third part consists of perturbative coefficient functions. The differential cross-section is determined by those twist-2- and twist-3 matrix elements of the initial hadrons and a forward hard scattering of partons from the initial hadrons. The perturbative coefficient functions describe the forward hard scattering. If the factorization can be proven, the coefficient functions can be calculated safely as a perturbative expansion and they are free from any soft divergence like collinear- and I.R. divergence. In this approach the effects of helicity-flip are parameterized with twist-3 matrix elements, while the absorptive part is generated in the hard scattering of partons.

The above mentioned collinear factorization has been derived in a rather formal way by using diagram expansion at hadron level, in which one divides diagrams into three parts. Two of them are related to the initial hadrons respectively, the remaining one is related to the parton scattering. By expanding the two parts related to hadrons according to twist of operators and the part for the parton scattering with large momentum transfers, one obtains the factorized form of SSA and the perturbative coefficient function at leading order of α_s . The forward hard scattering is participated by three partons from the polarized hadron, i.e., two quarks with one gluon and two partons from the unpolarized hadron, e.g., $\bar{q} + (q + g) \rightarrow \gamma^* + X \rightarrow \bar{q} + q$ with the antiquark from the unpolarized hadron and other partons from the polarized one. It seems difficult with this method to derive the perturbative coefficient function at higher orders and to prove the factorization. It is interesting to note that the contributions in the factorization consist of two parts. One part of the contributions is with the gluon carrying a nonzero momentum, called as hard-pole contributions, while another part of the contributions is with the gluon carrying zero momentum, called as soft-gluon-pole contributions. The two parts are associated with perturbative coefficient functions starting at the same leading order of α_s .

It should be noted that QCD factorizations, if they are proven, are general properties of QCD. These factorizations hold not only with hadron states but also hold when one replaces the hadron states with parton states. This is in the sense that the perturbatively calculable parts in factorizations do not depend on hadrons and are completely determined by hard scattering of partons. The procedure for this in the case of SSA is the following: With partonic states one can calculate the differential cross-section related to SSA and those twist-2 and twist-3 matrix elements with perturbative theory. In general they will contain soft divergences which usually appear beyond leading order. By writing the differential cross-section as a convolution of these matrix elements with a perturbative coefficient function, one can determine the function. Beyond the leading order of α_s , one may be able to show that the function is free from soft divergences. If it is true, then the factorization for SSA is proven. This procedure also provides a way to determine the higher-order corrections to the perturbative coefficient function.

In our previous works[13, 14, 15] we have made such an attempt to derive the factorization by replacing hadrons with partons. To have helicity-flip with massless partons we have constructed a multi-parton state to replace the transversely polarized hadron in [15]. But with our partonic results at leading order of α_s we can only find the hard-pole contributions. This is apparently in contradiction with the early results.

In factorizations only with leading twist-2 operators, it is interesting to note the following fact: For a differential cross-section the factorizations at leading order is completely determined by partonic results at tree-level, i.e., the perturbative coefficient functions at leading order are determined only with the

differential cross-section and twist-2 matrix elements calculated at tree-level with parton states. This has the implication for one-loop results. If the factorization is right or proven, then the collinearly divergent part of the differential cross-section with parton states at one-loop is completely determined by the convolutions of the leading-order perturbative coefficient functions with the one-loop matrix elements of twist-2 operators. This pattern about the collinearly divergent part of the differential cross-section can be iterated beyond one-loop. Assuming this is also the case for the factorization involving twist-3 operators, one then expects that the collinearly divergent part of the differential cross-section related to SSA at one-loop is completely determined by the convolution of the collinearly divergent parts of twist-2 and twist-3 matrices at one-loop with the perturbative coefficient functions at leading order. However, the assumptions may not be correct. Factorizations involving twist-3 operators can be different than those only with twist-2 operators.

In order to clarify this issue we go beyond the leading order in this work. We find that certain contributions at one-loop, which contain collinear divergences, can not be factorized in the way as expected in the above. To make the factorization with the partonic states correct, one has to introduce additional contributions in the derived factorization which only contains hard-pole contributions. These additional contributions are just the soft-gluon contributions. This is an interesting fact because a part of the leading-order perturbative coefficient function is determined by quantities at non-leading order. It will be important for calculating higher-order corrections and proving the factorization, following the above outlined procedure. In this work we restrict ourselves to the case where the forward hard scattering of those partons consisting of two antiquarks from the unpolarized hadron and two quarks with one gluon from the polarized hadron. Besides the above mentioned contributions we also find an extra hard-pole contribution corresponding to the forward hard scattering where only the gluon is in the initial- or final state. By calculating the twist-3 matrix element corresponding to this hard scattering, we can solve a part of discrepancies in the evolution of the matrix element studied in [16, 17, 18].

In this work we study SSA in Drell-Yan processes in the kinematic limit of the small transverse momentum of the observed lepton pair. In this kinematic region there exists another factorization for SSA, called Transverse-Momentum-Dependent(TMD) factorization, similarly to the TMD factorization for unpolarized cases studied in [21, 22, 23, 24, 25]. In the polarized case the nonperturbative effects of the polarized hadron are factorized into Sivers function[19] which contains both helicity-flip- and T-odd effects. The properties of Sivers function and SSA with it have been studied extensively [26, 27, 28, 29, 30, 31, 32, 33, 34, 35]. It should be noted that in the kinematic region of the small transverse momentum limit two factorizations apply, if the transverse momentum is much larger than the QCD scale Λ_{QCD} . It has been shown that the two factorizations are equivalent in the region[10, 11, 12]. Again, the TMD factorization here is derived in the formal way by using the mentioned diagram expansion at hadron level. In [13, 14, 15] we have examined the TMD factorization of SSA with parton states and found an agreement with existing results. In this work we will only focus on the collinear factorization.

Our work is organized as the following: In Sect.2 we give our notations for Drell-Yan processes and the factorization of SSA. In Sect.3 we introduce our multi-parton states and give some relevant results of twist-3 matrix elements. In Sect.4 we calculate SSA with our multi-parton state from certain classes of one-loop contributions which are collinearly divergent. We then show that these contributions can not be factorized as the usual way discussed in the above. These contributions in fact have to be identified as the mentioned soft-gluon-pole contributions in order to have finite corrections at higher orders of perturbative coefficient functions. In Sect. 5 we study the extra contribution which should be added to the factorization formula. There we also show that a part of the discrepancy in the evolution of the twist-3 matrix element derived in [16, 17] is solved. Sect.6 is our summary and outlook.

2. Collinear Factorization of SSA in Drell-Yan Processes

We will use the light-cone coordinate system, in which a vector a^μ is expressed as $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$ and $a_\perp^2 = (a^1)^2 + (a^2)^2$. Other notations are:

$$g_\perp^{\mu\nu} = g^{\mu\nu} - n^\mu l^\nu - n^\nu l^\mu, \quad \epsilon_\perp^{\mu\nu} = \epsilon^{\alpha\beta\mu\nu} l_\alpha n_\beta, \quad \epsilon^{\alpha\beta\mu\nu} = -\epsilon_{\alpha\beta\mu\nu}, \quad \epsilon^{0123} = 1 \quad (1)$$

with the light-cone vectors l and n defined as $l^\mu = (1, 0, 0, 0)$ and $n^\mu = (0, 1, 0, 0)$, respectively. We consider the Drell-Yan process:

$$h_A(P_A, s) + h_B(P_B) \rightarrow \gamma^*(q) + X \rightarrow \ell^- + \ell^+ + X, \quad (2)$$

where h_A is a spin-1/2 hadron with the spin-vector s . We take a light-cone coordinate system in which the momenta and the spin are :

$$P_{A,B}^\mu = (P_{A,B}^+, P_{A,B}^-, 0, 0), \quad s^\mu = (0, 0, \vec{s}_\perp). \quad (3)$$

h_A moves in the z -direction, i.e., P_A^+ is the large component. h_B moves in the $-z$ -direction with P_B^- as the large component. The spin of h_B is averaged. The invariant mass of the observed lepton pair is $Q^2 = q^2$. The relevant hadronic tensor is defined as a matrix element of the forward scattering $h_A + h_B \rightarrow \gamma^* + X \rightarrow h_A + h_B$:

$$W^{\mu\nu} = \sum_X \int \frac{d^4x}{(2\pi)^4} e^{iq \cdot x} \langle h_A(P_A, s_\perp), h_B(P_B) | \bar{q}(0) \gamma^\nu q(0) | X \rangle \langle X | \bar{q}(x) \gamma^\mu q(x) | h_B(P_B), h_A(P_A, s_\perp) \rangle, \quad (4)$$

and the differential cross-section is determined by the hadronic tensor as:

$$\frac{d\sigma}{dQ^2 d^2q_\perp dq^+ dq^-} = \frac{4\pi\alpha_{em}^2 Q_q^2}{3SQ^2} \delta(q^2 - Q^2) \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) W^{\mu\nu}, \quad S = 2P_A^+ P_B^-. \quad (5)$$

We are interested in the kinematical region where $q_\perp^2 \ll Q^2$. The hadronic tensor at leading twist accuracy has the structure:

$$\begin{aligned} W^{\mu\nu} = & -g_\perp^{\mu\nu} W_U^{(1)} + \left(g_\perp^{\mu\nu} - 2 \frac{q_\perp^\mu q_\perp^\nu}{q_\perp^2} \right) W_U^{(2)} \\ & - g_\perp^{\mu\nu} \epsilon_\perp^{\alpha\beta} s_{\perp\alpha} q_{\perp\beta} W_T^{(1)} + \left(s_{\perp\alpha} \epsilon_\perp^{\alpha\mu} q_\perp^\nu + s_{\perp\alpha} \epsilon_\perp^{\alpha\nu} q_\perp^\mu - g_\perp^{\mu\nu} \epsilon_\perp^{\alpha\beta} s_{\perp\alpha} q_{\perp\beta} \right) W_T^{(2)} \\ & + q_{\perp\alpha} (\epsilon_\perp^{\alpha\mu} q_\perp^\nu + \epsilon_\perp^{\alpha\nu} q_\perp^\mu) \vec{q}_\perp \cdot \vec{s}_\perp W_T^{(3)} + \dots \end{aligned} \quad (6)$$

In the above, we only give the structures symmetric in $\mu\nu$. $W_T^{(i)} (i = 1, 2, 3)$ represent T -odd effect related to the spin. $W_U^{(1,2)}$ are responsible for unpolarized cross-sections. $W_T^{(1)}$ contributes to SSA in the region $q_\perp^2 \ll Q^2$ which we will study. We introduce $q^+ = xP_A^+$ and $q^- = yP_B^-$. All structure functions depend on x , y and q_\perp^2 .

In the limit $q_\perp \rightarrow 0$ only the structure function $W_T^{(1)}$ gives the leading spin-dependent contribution to the differential cross-section and hence to SSA. The factorization of the structure function is the main subject to be studied in this work. In the collinear factorization $W_T^{(1)}$ is factorized with standard parton

distributions of hadron h_B and twist-3 matrix elements of hadron h_A . There are two relevant twist-3 matrix elements. They are defined as:

$$\begin{aligned}
T_F(x_1, x_2) \epsilon_{\perp}^{\mu\nu} s_{\perp\nu} &= \frac{g_s}{2} \int \frac{dy_1 dy_2}{4\pi} e^{-iy_2(x_2-x_1)P^+ - iy_1 x_1 P^+} \\
&\cdot \{ \langle P, \vec{s}_{\perp} | \bar{\psi}(y_1 n) \gamma^+ G^{+\mu}(y_2 n) \psi(0) | P, \vec{s}_{\perp} \rangle - (\vec{s}_{\perp} \rightarrow -\vec{s}_{\perp}) \}, \\
T_{\Delta,F}(x_1, x_2) s_{\perp}^{\mu} &= -i \frac{g_s}{2} \int \frac{dy_1 dy_2}{4\pi} e^{-iy_2(x_2-x_1)P^+ - iy_1 x_1 P^+} \\
&\cdot \{ \langle P, \vec{s}_{\perp} | \bar{\psi}(y_1 n) \gamma^+ \gamma_5 G^{+\mu}(y_2 n) \psi(0) | P, \vec{s}_{\perp} \rangle - (\vec{s}_{\perp} \rightarrow -\vec{s}_{\perp}) \}. \tag{7}
\end{aligned}$$

In the above we have suppressed the gauge links along direction n between operators. These gauge links make the definition gauge invariant. The general properties of these twist-3 matrix elements have been discussed in [4, 7]. One can show $T_{\Delta,F}(x, x) = 0$. In general $T_F(x, x)$ is not zero. This corresponds to the so-called soft-gluon-pole contributions because the gluon field in $T_F(x_1, x_2)$ with $x_1 = x_2 = x$ carries zero momentum entering the hard scattering.

For the case that SSA is generated through the scattering where an antiquark \bar{q} from the unpolarized hadron h_B and a quark q or a quark with a gluon from the polarized hadron h_A , i.e, the forward parton scattering $\bar{q} + q + g \rightarrow \gamma^* + X \rightarrow \bar{q} + q$ or the reversed, the structure function in the limit $q_{\perp}/Q \ll 1$ can be factorized in the form[10]:

$$\begin{aligned}
W_T^{(1)}(x, y, q_{\perp}) &= \frac{\alpha_s}{(2\pi q_{\perp}^2)^2} \int_x^1 \frac{dy_1}{y_1} \int_y^1 \frac{dy_2}{y_2} \bar{q}(y_2) \left[\mathcal{A}_h(x, y_1) + \mathcal{A}_s(x, y_1) + \mathcal{A}_{sq}(x, y_1) \right], \\
\mathcal{A}_h(x, y_1) &= \delta(1 - \xi_2) \left[T_F(x, y_1) \frac{1 + \xi_1}{(1 - \xi_1)_+} + T_{\Delta,F}(x, y_1) \right], \\
\mathcal{A}_s(x, y_1) &= \frac{1}{N_c^2} \left[y_1 \frac{\partial T_F(y_1, y_1)}{\partial y_1} (1 + \xi_1^2) \delta(1 - \xi_2) + T_F(y_1, y_1) \left(\frac{2\xi_1^3 - 3\xi_1^2 - 1}{(1 - \xi_1)_+} \delta(1 - \xi_2) \right. \right. \\
&\quad \left. \left. - \frac{\xi_2(1 + \xi_2^2)}{(1 - \xi_2)_+} \delta(1 - \xi_1) + 2\delta(1 - \xi_1) \delta(1 - \xi_2) \ln \frac{q_{\perp}^2}{Q^2} \right) \right], \\
\mathcal{A}_{sq}(x, y_1) &= T_F(y_1, y_1) \delta(1 - \xi_1) \left[\left(1 + \frac{\xi_2 - 1}{N_c^2} \right) \frac{1 + \xi_2^2}{(1 - \xi_2)_+} - 2\delta(1 - \xi_2) \ln \frac{q_{\perp}^2}{Q^2} \right], \\
\xi_1 &= x/y_1, \quad \xi_2 = y/y_2. \tag{8}
\end{aligned}$$

In the above $\bar{q}(y_2)$ is the antiquark distribution function of h_B . The contributions to $W_T^{(1)}$ can be divided into three parts. The part with \mathcal{A}_h consists of the hard-pole contributions. In \mathcal{A}_h , the first term with $T_F(x_1, x_2)$ has been first derived in [10] which is also confirmed in [15], while the second term with $T_{\Delta,F}$ has been derived in [15]. The part with \mathcal{A}_s and that with \mathcal{A}_{sq} are of the soft-gluon-pole contributions, because they are related to $T_F(x, x)$. The soft-gluon-pole contributions in \mathcal{A}_{sq} only appear in the limit in the limit $Q \gg q_{\perp}$.

3. Multi-parton State and Twist-3 Matrix elements

In [13, 14] we have studied SSA with single-parton states, where the helicity flip is caused by a finite quark mass m . In fact one can study SSA at parton level by taking massless limit $m = 0$. For this one needs to take the effect of helicity flip as that of a correlation between the spin of quarks and the spin of

gluons. For this purpose we consider the state or the system with the total helicity λ :

$$|n[\lambda]\rangle = |q(p, \lambda_q)[\lambda]\rangle + c_1 |q(p_1, \lambda_q)g(k, \lambda_g)[\lambda]\rangle, \quad (9)$$

with $p_1 + k = p$. In the first term $\lambda_q = \lambda$. For the qg -state, the total helicity is the sum $\lambda_q + \lambda_g$. We specify the momentum as:

$$p^\mu = (p^+, 0, 0, 0), \quad p_1^\mu = x_0 p^\mu, \quad k^\mu = (1 - x_0) p^\mu = \bar{x}_0 p^\mu. \quad (10)$$

The q -state and qg -state carries the same color index i_c as given:

$$|q(p, \lambda_q)\rangle = b_{i_c}^\dagger(p, \lambda_q)|0\rangle, \quad |q(p_1, \lambda_q)g(k, \lambda_g)\rangle = T_{j_c i_c}^a b_{j_c}^\dagger(p_1, \lambda_q) a_a^\dagger(k, \lambda_g)|0\rangle, \quad (11)$$

where b_i^\dagger is the quark creation operator with i as the color index, a_a^\dagger is the gluon creation operator with a as the color index. c_1 is taken as a real number.

From standard text book we know that the transverse-spin dependent part of a matrix element, like the twist-3 matrix elements or the transverse-spin dependent part of $W^{\mu\nu}$, corresponds to the off-diagonal part of the matrix element in helicity space. Because of helicity conservation in QCD with massless quarks, the twist-3 matrix elements are always zero if we replace the hadron in the matrix elements with a single quark. However, if one replaces the hadron with the above multi-parton state, the twist-3 matrix elements receive nonzero contributions from the interference between the single quark state and the state consisting of a quark and a gluon. In the interference, the quark always has the same helicity, while the helicity change is due to the helicity of the gluon. The structure function $W_T^{(1)}$ also receives nonzero contributions from the interference.

By replacing the hadron with the multi-parton state one can calculate those twist-3 matrix elements and the structure functions perturbatively for the purpose of factorization. At tree-level, it is straightforward to obtain the twist-3 matrix elements as:

$$\begin{aligned} T_F(x_1, x_2) &= c_1 g_s \pi \sqrt{\frac{x_0}{2}} (N_c^2 - 1) (x_2 - x_1) [\delta(1 - x_1) \delta(x_2 - x_0) - \delta(1 - x_2) \delta(x_1 - x_0)], \\ T_{\Delta, F}(x_1, x_2) &= c_1 g_s \pi \sqrt{\frac{x_0}{2}} (N_c^2 - 1) (x_2 - x_1) [\delta(1 - x_1) \delta(x_2 - x_0) + \delta(1 - x_2) \delta(x_1 - x_0)]. \end{aligned} \quad (12)$$

These functions are proportional to c_1 indicating that they are from the mentioned interference. For simplicity we will set $c_1 = 1$ in the following sections without confusion.

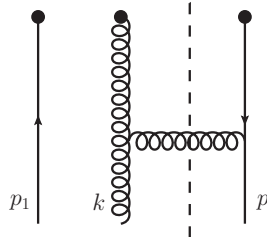


Figure 1: A diagram for the twist-3 matrix element $T_F(x_1, x_2)$ which gives a nonzero contribution with $x_1 = x_2 = x$. Black dots denote insertion of operators in the definition of $T_F(x_1, x_2)$. The broken line is a cut.

It is noted that at tree-level the function $T_F(x, x)$ becomes zero. This is the reason why we can not re-produce in [15] the soft-gluon-pole contributions with tree-level results of T_F and $W_T^{(1)}$. However, the function becomes nonzero at one-loop. To show this, we examine a particular contribution from a one-loop diagram given in Fig.1 in Feynman gauge. The contributions contain a U.V. divergence and a collinear divergence. Regularizing these divergences and subtracting the U.V. one we have:

$$\begin{aligned}
T_F(x_1, x_2) \Big|_{Fig.1} &= \frac{g_s \alpha_s}{16} N_c (N_c^2 - 1) \sqrt{2x_0} \left[\delta(x_2 - x_0) \frac{1 - x_1}{1 - x_0} \left(\frac{x_0 - x_1}{1 - x_1} - 2x_0 \right) \right. \\
&\quad \left. + \delta(x_1 - x_0) \frac{1 - x_2}{1 - x_0} \left(\frac{x_0 - x_2}{1 - x_2} - 2x_0 \right) \right] \left(-\frac{2}{\epsilon_c} + \gamma - \ln \frac{\mu^2}{4\pi\mu_c^2} \right), \\
T_{\Delta,F}(x_1, x_2) \Big|_{Fig.1} &= \frac{g_s \alpha_s}{16} N_c (N_c^2 - 1) \sqrt{2x_0} \left[\delta(x_2 - x_0) \frac{1 - x_1}{1 - x_0} \left(\frac{x_0 - x_1}{1 - x_1} - 2x_0 \right) \right. \\
&\quad \left. - \delta(x_1 - x_0) \frac{1 - x_2}{1 - x_0} \left(\frac{x_0 - x_2}{1 - x_2} - 2x_0 \right) \right] \left(-\frac{2}{\epsilon_c} + \gamma - \ln \frac{\mu^2}{4\pi\mu_c^2} \right), \quad (13)
\end{aligned}$$

where the pole in $\epsilon_c = 4 - d$ represents the collinear divergence and μ_c is the scale associated with it. The scale μ is associated with the subtracted U.V. divergence. The collinear divergence appears because the gluon going through the cut can be collinear to the incoming gluon and the outgoing quark in Fig.1. In the above we also give the contribution to $T_{\Delta,F}$ from Fig.1. From this result we see that T_F is nonzero at $x_1 = x_2$. After examining all one-loop diagrams in Feynman gauge, we find that only the diagram in Fig.1 gives nonzero contribution at $x_1 = x_2$, i.e.,

$$T_F(x, x) = -\frac{g_s \alpha_s}{4} N_c (N_c^2 - 1) x_0 \sqrt{2x_0} \delta(x_0 - x) \left(-\frac{2}{\epsilon_c} + \gamma - \ln \frac{\mu^2}{4\pi\mu_c^2} \right). \quad (14)$$

This result is in agreement with our previous calculation in the light-cone gauge $n \cdot G = 0$ [15]. The function $T_{\Delta,F}$ is always zero at $x_1 = x_2 = x$.

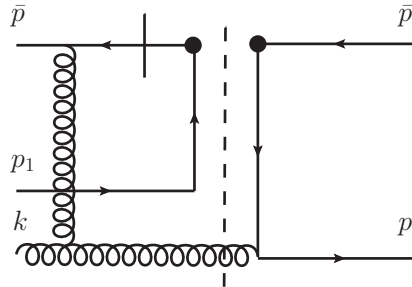


Figure 2: The diagram for the tree-level contribution to SSA in the limit $q_\perp \ll Q$. The black dots represent the insertion of electro-magnetic current operators. The broken line is the cut. The short bar cutting the quark propagator means to take the absorptive part of the propagator.

To calculate the hadronic tensor, we replace the polarized hadron h_A with the parton state $|n\rangle$ of Eq.(9), and the unpolarized hadron h_B with an antiquark with the momentum $\bar{p}^\mu = (0, \bar{p}^-, 0, 0)$. At tree-level, $W_T^{(1)}$ also receives contributions from the interference. i.e., from the forward scattering $\bar{q} + q + g \rightarrow \gamma^* + g \rightarrow \bar{q} + q$ and $\bar{q} + q \rightarrow \gamma^* + g \rightarrow \bar{q} + q + g$. In the limit $q_\perp \ll Q$, only one diagram

given in Fig.2 gives the contribution. In the diagram the short bar means to take the absorptive part of the cut propagator with the momentum k_q , i.e.,

$$\text{Abs} \left[\frac{i\gamma \cdot k_q}{k_q^2 + i\varepsilon} \right] = i\gamma \cdot k_q \left(-i\pi\delta(k_q^2) \right). \quad (15)$$

In fact, the short bar here represents a physical cut of the amplitude of the left part in the diagram. It has been shown in [15] with the tree-level results of the twist-3 matrix elements given in Eq.(12) that the tree-level result of $W_T^{(1)}$ only produces the \mathcal{A}_h -term in Eq.(8). If one expects that the factorization involving twist-3 operators here happens in the same way as in the factorization only with twist-2 operators, one will conclude that at leading order of α_s of the perturbative coefficient function $W_T^{(1)}$ is predicted only with the hard-pole contributions of Eq.(8). This is obvious in contradiction with the results in Eq.(8).

4. Soft-Gluon-Pole Contributions

At tree-level, all momenta carried by gluon lines in Fig.2 are fixed and can not be zero, e.g., the momentum k_1 of the gluon crossing the cut is fixed by the total momentum conservation. Therefore, we can not identify any gluon line in Fig.2 corresponding to the gluon field with zero momentum in $T_F(x, x)$ in Eq.(7). Now we consider the case in which there is an extra gluon exchanged and crossing the cut as those diagrams given in Fig.3. In this case the momentum k_1 has to be integrated. In the integration the collinear region, in which the gluon with k_1 is collinear to the incoming gluon and outgoing quark, is included. This will result in a collinear divergence. Taking Fig.3a as such an example, there is an extra gluon exchanged in comparison with Fig.2. The momentum k_1 hence will be integrated. In the collinear region of k_1 , one can realize that the part of Fig.3a including the three gluon vertex and the vertex absorbing the gluon with k_1 is essentially given by Fig.1. This indicates that the collinearly divergent part from the collinear region may be factorized with $T_F(x_1, x_2)$ or $T_{\Delta, F}$ given in Fig.1 in the form of a convolution of T_F or $T_{\Delta, F}$ with a perturbative coefficient function. If the function is not the same as those in \mathcal{A}_h of Eq.(8) determined at leading order, then one has to add extra terms beside the term with \mathcal{A}_h in Eq.(8) to make sure that the factorization is correct at one-loop level. It is interesting to note that by taking the absorptive part of the quark propagator in the left part of Fig.3a, one finds that in the collinear region of k_1 the momentum of the gluon exchanged between the initial gluon and initial \bar{q} is a soft gluon. More precisely, the exchanged gluon is a Glauber gluon with the momentum pattern $k^\mu \sim (\lambda_0^2, \lambda_0^2, \lambda_0, \lambda_0)$ with $\lambda_0 \ll 1$. This indicates that the possible extra terms may be soft-gluon-pole contributions factorized with $T_F(x, x)$ from Fig.1. In this section we show that this is indeed the case.

As discussed in the above, it is easy to find those diagrams at one-loop which can give contributions to the soft-gluon-pole contributions. The diagrams are those where the incoming gluon emits a collinear gluon and the collinear gluon is absorbed by an outgoing collinear quark. Beside these gluons, there is an extra exchanged gluon crossing the cut responsible for the finite q_\perp . A class of those diagrams is given in Fig.3. For our purpose we consider those contributions from the collinear region where the gluon crossing the cut and emitted by the initial gluon is collinear to the initial gluon and the outgoing quark.

We take Fig.3a as an example to show how we obtain the collinearly divergent part. The contribution from Fig.3a to the hadronic tensor can be written with standard Feynman rule as:

$$\begin{aligned} W^{\mu\nu} \Big|_{3a} = & -\frac{ig_s^5}{2N_c} f^{a_1 a_3 a} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \delta^4(\bar{p} + p - k_1 - k_2 - q) (2\pi\delta(k_1^2)) (2\pi\delta(k_2^2)) (-i\pi\delta((k_3 + \bar{p})^2)) \\ & \cdot \left[\bar{v}(\bar{p}) \gamma^{\mu_3} \gamma \cdot (k_3 + \bar{p}) \gamma^\mu \gamma \cdot (p_1 - k_2) \gamma^{\rho_2} T^{a_3} T^{a_2} T^a u(p_1, \lambda_q) \right] \end{aligned}$$

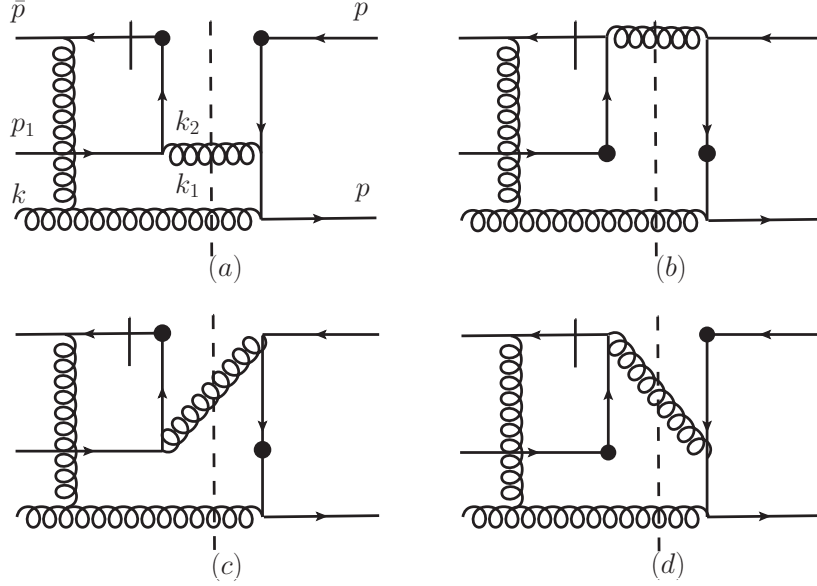


Figure 3: The diagrams for the amplitude $\bar{q} + q + G \rightarrow \gamma^* + G + G \rightarrow \bar{q} + q$, which gives a part of contributions to SSA.

$$\begin{aligned}
& \cdot \bar{u}(p, \lambda_q) T^{a_1} T^{a_2} \gamma_{\rho_1} \gamma \cdot (p - k_1) \gamma_{\rho_2} \gamma \cdot (p - k_1 - k_2) \gamma^\nu v(\bar{p}) \\
& \left(\epsilon \cdot (-k_1 + k_3) g^{\rho_1}_{\mu_3} + (-k_3 - k)^{\rho_1} \epsilon_{\mu_3} + (k_1 + k)_{\mu_3} \epsilon^{\rho_1} \right) \Big] \\
& \frac{1}{((p - k_1 - k_2)^2 - i\varepsilon)((p - k_1)^2 - i\varepsilon)((p_1 - k_2)^2 + i\varepsilon)(k_3^2 + i\varepsilon)},
\end{aligned} \tag{16}$$

the color and spin of the initial antiquark \bar{q} is averaged and it gives the factor $(2N_c)^{-1}$. The initial quark has the helicity λ_q , the initial gluon has λ_g . The absorptive part in the scattering amplitude is generated by the cut cutting the quark propagator. This gives the $\delta((k_3 + \bar{p})^2)$ with k_3 being the momentum carried by the gluon exchanged between the initial gluon and the initial antiquark in the left part of Fig.3a. We will consider the collinear region where the momentum k_1 of the gluon emitted by the initial gluon is collinear. In the collinear region the momentum k_1 scales as:

$$k_1^\mu \sim (1, \lambda_0^2, \lambda_0, \lambda_0) \tag{17}$$

with $\lambda_0 \ll 1$. The on-shell condition from the quark propagator fixes k_1^+ in the collinear region as

$$\frac{1}{(k_3 + \bar{p})^2 + i\varepsilon} \Rightarrow -i\pi\delta((k_3 + \bar{p})^2) \Rightarrow k_1^+ \approx k^+ - \frac{k_{1\perp}^2}{2\bar{p}^-} + \mathcal{O}(\lambda_0^4). \tag{18}$$

This constraint leads to that the gluon with k_3 is a Glauber gluon. It is soft and may be represented by the gluon field in $T_F(x, x)$.

The evaluation of the contribution containing the collinear divergence from the collinear region given by Eq.(17) is rather straightforward. One first uses these δ -functions to perform the integration over k_2 , k_1^- and k_1^\perp . The remaining integration is that over $k_{1\perp}$. The integrand is then besides some trivial factors a product of those terms in $[\dots]$ in Eq.(16), the denominators of propagators and the $\delta(k_2^2)$.

Now one can expand the integrand in λ_0 . The leading order is at λ_0^{-3} which does not give the collinear divergence. The next-to-leading order is at λ_0^{-2} , which give the collinear divergence after the integration over $k_{1\perp}$. Contributions from higher orders are finite. In the expansion we notice that the δ -function $\delta(k_2^2)$ also depends on k_1 and needs to be expanded. The expansion will give a contribution proportional to the derivative of the δ -function. This contribution may correspond to those terms in Eq.(8) with the derivative of $T_F(x, x)$.

After the integration over $k_{1\perp}$ one can take the limit $q_\perp \ll Q$. To derive the limit we will use the following in the limit of $q_\perp \rightarrow 0$:

$$\begin{aligned} s\delta(s(1-y)(x_0-x)-q_\perp^2) &\approx \frac{\delta(1-y)}{(x_0-x)_+} + \frac{\delta(x_0-x)}{(1-y)_+} - \delta(x_0-x)\delta(1-y)\ln\frac{q_\perp^2}{Q^2}, \\ s\delta'(s(1-y)(x_0-x)-q_\perp^2) &\approx \frac{1}{q_\perp^2}\delta(x_0-x)\delta(1-y), \quad \delta'(u) = \frac{d\delta(u)}{du}, \end{aligned} \quad (19)$$

with $s = 2p^+\bar{p}^-$. The calculation can be simplified by the following: We need only to calculate the off-diagonal part of the matrix element in helicity space. In this part one always has $\lambda_q\lambda_g = -1$. We will set $\lambda_q\lambda_g = -1$ in our calculation for simplicity. We have the collinearly divergent part of the hadronic tensor in the limit $q_\perp \ll Q$:

$$W^{\mu\nu}\Big|_{3a} = -i\frac{g_s\alpha_s^2}{(4\pi)^2}\frac{N_c^2-1}{N_c}g_\perp^{\mu\nu}\frac{\vec{\epsilon}(\lambda_g)\cdot\vec{q}_\perp}{(q_\perp^2)^2}\sqrt{x_0}\frac{x_0^2-x^2}{x_0^2}\delta(1-y)\left(-\frac{2}{\epsilon_c}\right)+\dots \quad (20)$$

where \dots stand for the following contributions: the contributions at non-leading order in the expansion in q_\perp/Q , the contributions which do not contain the collinear divergence and the contributions of tensor structures other than $g_\perp^{\mu\nu}$. These contributions are irrelevant for our purpose. By adding the complex conjugated contribution with different parton helicities as a part of the interference, one can then obtain the off-diagonal part of the hadronic tensor in the helicity space. From the off-diagonal part we can extract the structure function:

$$W_T^{(1)}\Big|_{3a} = \frac{g_s\alpha_s^2}{(4\pi)^2}\frac{N_c^2-1}{N_c}\frac{1}{(q_\perp^2)^2}\sqrt{2x_0}\frac{x_0^2-x^2}{x_0^2}\delta(1-y)\left(-\frac{2}{\epsilon_c}\right)+\dots, \quad (21)$$

again, \dots denote those irrelevant contributions which do not contain the collinear divergence or are not at the leading order in the expansion in q_\perp/Q . In the limit the leading order of $W_T^{(1)}$ is of q_\perp^{-4} . In the following we will only give the collinearly divergent contributions in the limit of $q_\perp \rightarrow 0$ explicitly. Performing similar calculations we have the contributions from other diagrams in Fig.3:

$$\begin{aligned} W_T^{(1)}\Big|_{3b} &= \mathcal{O}(q_\perp^{-2}), \\ W_T^{(1)}\Big|_{3c} &= \frac{g_s\alpha_s^2}{(4\pi)^2}\frac{N_c^2-1}{N_c}\left(-\frac{2}{\epsilon_c}\right)\frac{\sqrt{2x_0}}{(q_\perp^2)^2}\left\{\frac{x_0y\delta(x_0-x)}{(1-y)_+} + \frac{x\delta(1-y)}{(x_0-x)_+} \right. \\ &\quad \left. + x_0\delta(x_0-x)\delta(1-y)\left(1-\ln\frac{q_\perp^2}{Q^2}\right)\right\}+\dots, \\ W_T^{(1)}\Big|_{3d} &= \frac{g_s\alpha_s^2}{(4\pi)^2}\frac{N_c^2-1}{N_c}\left(-\frac{2}{\epsilon_c}\right)\frac{\sqrt{2x_0}}{(q_\perp^2)^2}\left\{\frac{x_0y^2\delta(x_0-x)}{(1-y)_+} + \frac{x^2\delta(1-y)}{x_0(x_0-x)_+} \right. \\ &\quad \left. + x_0\delta(x_0-x)\delta(1-y)\left(1-\ln\frac{q_\perp^2}{Q^2}\right)\right\}+\dots, \end{aligned} \quad (22)$$

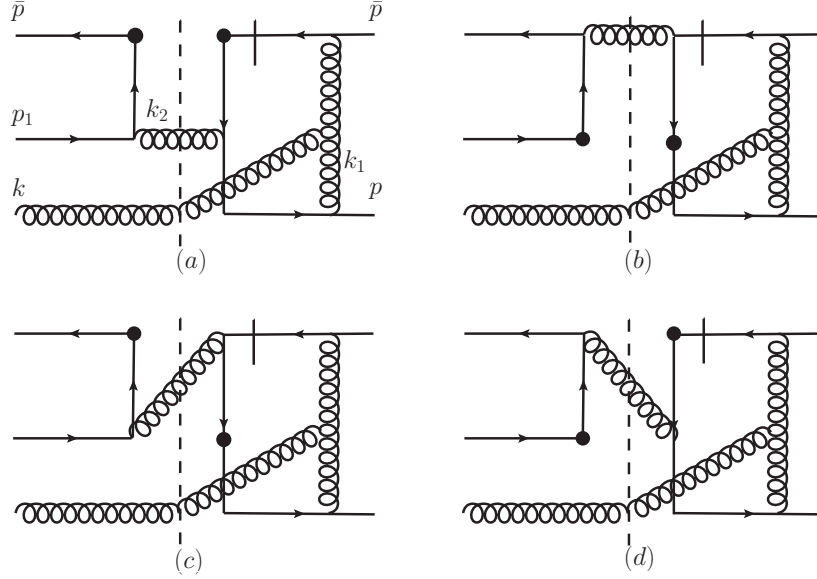


Figure 4: The diagrams for the amplitude $\bar{q} + q + G \rightarrow \gamma^* + G + G \rightarrow \bar{q} + q$, which gives a part of contributions to SSA.

Beside those diagrams in Fig.3, there is another class of diagrams which give the wanted contributions. The calculations of these diagrams are slightly different than those of Fig.3. We illustrate this by taking Fig.4a as an example. The contribution from Fig.4a can be written as:

$$\begin{aligned}
 W^{\mu\nu} \Big|_{4a} = & -\frac{g_s^5}{2N_c} f^{a_1 a_3 a} \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \delta^4(\bar{p} + p_1 - k_2 - q) (2\pi \delta(k_2^2)) (i\pi \delta((k_3 + \bar{p})^2)) \\
 & \left[\bar{u}(p) \gamma^{\rho_1} \gamma \cdot (p - k_1) \gamma^{\rho_2} \gamma \cdot (p - k_1 - k_2) \gamma^\nu \gamma \cdot (\bar{p} + k_3) \gamma^{\mu_3} T^{a_1} T^{a_2} T^{a_3} v(\bar{p}) \right. \\
 & \left. \cdot \bar{v}(\bar{p}) \gamma^\mu \gamma \cdot (p_1 - k_2) \gamma_{\rho_2} T^{a_2} T^a u(p_1) (\epsilon \cdot (-k_1 - k_3) g_{\rho_1 \mu_3} + (k_3 - k)_{\rho_1} \epsilon_{\mu_3} + (k_1 + k)_{\mu_3} \epsilon_{\rho_1}) \right] \\
 & \cdot \frac{1}{((p - k_1 - k_2)^2 - i\varepsilon)((p - k_1)^2 - i\varepsilon)((p_1 - k_2)^2 + i\varepsilon)(k_3^2 - i\varepsilon)(k_1^2 - i\varepsilon)}. \quad (23)
 \end{aligned}$$

Again, the initial quark has the helicity λ_q , the gluon has λ_g . The cut of the quark propagator gives the δ -function $\delta((k_3 + \bar{p})^2)$ with $k_3 = k_1 - k$ as the momentum of the gluon emitted by the antiquark in the right part of Fig.4a. k_1 is the momentum of the gluon emitted by the quark in the right part of Fig.4a. Unlike the gluon with k_1 in Fig.3a, where it is on-shell, the gluon with k_1 in Fig.4a is off-shell in general. This difference results in that the integration over k_1^- looks nontrivial at the first step, while the integration over k_1^- in Fig.3a can be simply done with the on-shell condition $\delta(k_1^2)$.

Now we consider the collinear region where k_1 is collinear to k and p . The scaling of its each component is given in Eq.(17). Then from the δ -function $\delta((k_3 + \bar{p})^2)$ k_1^+ is fixed as $k_1^+ = k^+ + \mathcal{O}(\lambda_0^2)$ after the integration over k_1^+ . The integration over k_1^- can be done with a contour in the complex k_1^- -plan. With the fixed k_1^+ one can show from Eq.(23) that there are poles from denominators of quark propagators only in the lower-half plan. These poles are corresponding physical cuts. One can use these poles by taking a contour in the lower-half plan to perform the integration. However, we notice that there

is only one pole in the upper-half plan. The pole is from the gluon propagator with the momentum k_1 . One can equivalently use this pole by taking a contour in the upper-half plan to perform the integration. Therefore, the integration over k_1^- can be done easily by the replacement:

$$\frac{1}{k_1^2 - i\varepsilon} \rightarrow 2\pi i \delta(k_1^2). \quad (24)$$

This also applies for other three diagrams in Fig.4. It is interesting to note that the gluon with k_1 or with k_3 in Fig.4a are in corresponding to the gluon with k_1 or with k_3 in Fig.3a, respectively. The gluon with k_3 in Fig.4a is also a Glauber gluon. The remaining calculations are similar to those of Fig.3. We have the following results of $W_T^{(1)}$ from Fig.4:

$$\begin{aligned} W_T^{(1)}|_{4a} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c^2 - 1}{N_c} \frac{\sqrt{2x_0}}{x_0^2} \frac{1}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c}\right) x(x_0 - x) \delta(1 - y), \\ W_T^{(1)}|_{4b} &= \frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c^2 - 1}{N_c} \sqrt{2x_0} \frac{1}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c}\right) x_0 y (1 - y) \delta(x_0 - x), \\ W_T^{(1)}|_{4c} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c^2 - 1}{N_c} \sqrt{2x_0} \frac{1}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c}\right) x_0 y \delta(x_0 - x), \\ W_T^{(1)}|_{4d} &= \frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c^2 - 1}{N_c} \frac{\sqrt{2x_0}}{x_0} \frac{1}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c}\right) x \delta(1 - y). \end{aligned} \quad (25)$$

Summing the contributions from Fig.3 and Fig.4 together we obtain the collinearly divergent contribution of $W_T^{(1)}$, denoted as $W_{T,s}^{(1)}$:

$$\begin{aligned} W_{T,s}^{(1)} &= \frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c^2 - 1}{N_c} \left(-\frac{2}{\epsilon_c}\right) \frac{\sqrt{2x_0}}{(q_\perp^2)^2} \left[2\delta(1 - \zeta) \delta(1 - y) \left(1 - \ln \frac{q_\perp^2}{Q^2}\right) \right. \\ &\quad \left. + \delta(1 - \zeta) \frac{y(1 + y^2)}{(1 - y)_+} - \delta(1 - y) \left(\frac{2\zeta^3 - 3\zeta^2 - 1}{(1 - \zeta)_+} + 2\zeta^2\right) \right], \\ \zeta &= \frac{x}{x_0}. \end{aligned} \quad (26)$$

If the factorization here takes the same patten as those only with twist-2 operators as discussed in the introduction and assuming that there is only the hard-pole contribution at leading order, one then expects that the above $W_{T,s}^{(1)}$ should obey:

$$W_{T,s}^{(1)} = \frac{\alpha_s}{(2\pi q_\perp^2)^2} \delta(1 - y) \int_x^1 \frac{dy_1}{y_1} \left[\frac{1 + \xi_1}{(1 - \xi_1)_+} T_F(x, y_1) \Big|_{Fig.1} + T_{\Delta,F}(x, y_1) \Big|_{Fig.1} \right] \quad (27)$$

so that the collinear divergences caused by the collinear gluon in Fig.3 and Fig.4 will not appear in the perturbative coefficient functions at one-loop. In the above we have already taken the tree-level result $\bar{q}(y_2) = \delta(1 - y_2)$. It is easy to see that the above equation does not hold because the color factor of $W_{T,s}^{(1)}$ does not match that of $T_F(x, y_1)$ and $T_{\Delta,F}(x, y_1)$ from Fig.1. Therefore, the factorization at leading order must contain extra terms besides the hard-pole contributions.

With the discussion at the beginning of this section, parts in each diagram in Fig.3 and Fig.4 can be identified with Fig.1. By deleting these parts, these one-loop diagrams reduce to those for the forward scattering $\bar{q} + q^* + g^* \rightarrow \gamma^* + X \rightarrow \bar{q} + q^*$. In the kinematic region considered here, the off-shell quark can

be approximately taken as an on-shell quark. The virtual gluon is the mentioned Glauber gluon. It carries the momentum with the $+$ -component approaching to zero. Therefore, $W_{T,s}^{(1)}$ should be factorized with $T_F(x, x)$ from Fig.1. With this observation, we realize that the factorization of this part is exactly given by the term with \mathcal{A}_s of Eq.(8). It is interesting to note that the term with the derivative of $T_F(x, x)$ is also reproduced. Therefore, we can already conclude here that factorizations involving twist-3 operators are different than those factorizations only with twist-2 operators. The perturbative coefficient function in a factorization involving twist-3 operators can not be completely determined by tree-level results of the differential cross-section and twist-3 matrix elements.

Besides the contributions studied in the above there are soft-gluon-pole contributions appearing in the limit $q_\perp \rightarrow 0$. There are two classes of diagrams given in Fig.5 and Fig.6. Their calculations are similar to those in Fig.3 and Fig.4. We only list their divergent contributions:

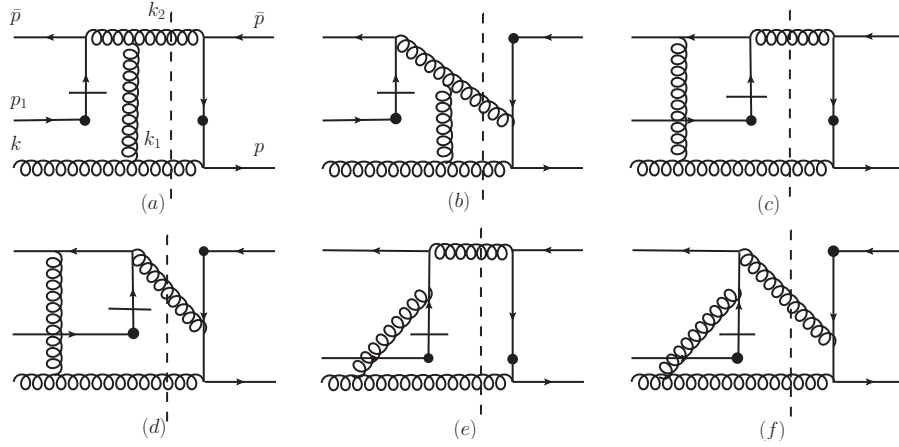


Figure 5: The diagrams for soft-gluon-pole contributions appearing in the limit $q_\perp \rightarrow 0$

$$\begin{aligned}
W_T^{(1)}|_{5a} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} N_c (N_c^2 - 1) \frac{1}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c}\right) \sqrt{2x_0} x_0 \delta(x - x_0) y + \dots, \\
W_T^{(1)}|_{5d} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{(N_c^2 - 1)}{N_c} \left(-\frac{2}{\epsilon_c}\right) \frac{x_0 \sqrt{2x_0}}{(q_\perp^2)^2} \delta(x - x_0) \left[\frac{y^2}{(1-y)_+} - \delta(1-y) \ln \frac{q_\perp^2}{Q^2} \right], \\
W_T^{(1)}|_{5e} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{(N_c^2 - 1)^2}{N_c} \left(-\frac{2}{\epsilon_c}\right) \frac{x_0 \sqrt{2x_0}}{(q_\perp^2)^2} \delta(x - x_0) (1-y), \\
W_T^{(1)}|_{5f} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{(N_c^2 - 1)^2}{N_c} \left(-\frac{2}{\epsilon_c}\right) \frac{x_0 \sqrt{2x_0}}{(q_\perp^2)^2} \delta(x - x_0) \left[\frac{y}{(1-y)_+} - \delta(1-y) \ln \frac{q_\perp^2}{Q^2} \right], \quad (28)
\end{aligned}$$

the contributions from Fig.5b and Fig.5c are non-leading in the limit of $q_\perp/Q \ll 1$.

The contributions from Fig.6 are:

$$\begin{aligned}
W_T^{(1)}|_{6a} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c (N_c^2 - 1)}{2} \frac{\sqrt{2x_0}}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c}\right) \left[\frac{\delta(x_0 - x)}{(1-y)_+} x_0 y (2y - 1) \right. \\
&\quad \left. + \frac{\delta(1-y)}{(x_0 - x)_+} \frac{x^2}{x_0} - x_0 \delta(x_0 - x) \delta(1-y) \ln \frac{q_\perp^2}{Q^2} \right],
\end{aligned}$$

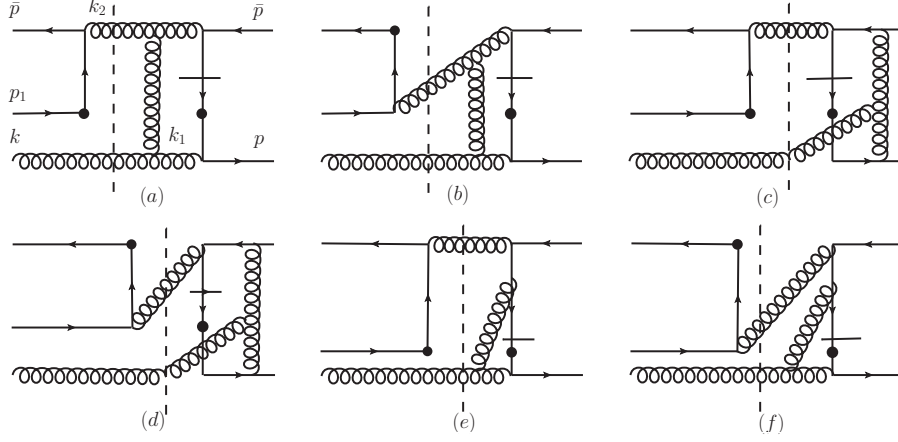


Figure 6: The diagrams for soft-gluon-pole contributions appearing in the limit $q_\perp \rightarrow 0$

$$\begin{aligned}
W_T^{(1)} \Big|_{6b} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c(N_c^2 - 1)}{2} \frac{\sqrt{2x_0}}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c} \right) \left[\frac{\delta(x_0 - x)}{(1-y)_+} x_0 y \right. \\
&\quad \left. + \frac{\delta(1-y)}{(x_0 - x)_+} \frac{x(2x_0 - x)}{x_0} - x_0 \delta(x_0 - x) \delta(1-y) \ln \frac{q_\perp^2}{Q^2} \right], \\
W_T^{(1)} \Big|_{6c} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c^2 - 1}{N_c} \frac{\sqrt{2x_0}}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c} \right) x_0 y (1-y) \delta(x_0 - x), \\
W_T^{(1)} \Big|_{6d} &= \frac{g_s \alpha_s^2}{(4\pi)^2} \frac{N_c^2 - 1}{N_c} \frac{\sqrt{2x_0}}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c} \right) x_0 y \delta(x_0 - x),
\end{aligned} \tag{29}$$

the contributions from Fig.6e and Fig.6f are non-leading in the limit of $q_\perp/Q \ll 1$. In Fig. 6a the gluon with k_1 corresponds to the gluon with k_1 in Fig.4a. We have observed that the integration over k_1^- here in Fig.6a is different that in Fig.4a. In the lower-half complex k_1^- -plan there is only one pole from the quark propagator in the right part of Fig.6a, and there are three poles from the three gluon propagators in the upper-half complex k_1^- -plan. One may take a contour in the lower-half complex k_1^- -plan to perform the k_1^- -integration whose result is only from the pole of the quark propagator. By taken a contour in the upper-half complex k_1^- -plan, this integration result can also be written a sum of contributions from the three poles of the gluon propagators. In the limit $q_\perp/Q \ll 1$, the terms in $W_T^{(1)}$ proportional to $\delta(x_0 - x)$ only come from the contribution of the pole in the gluon propagator with k_1 , i.e., these terms can equivalently be calculated with the k_1^- -integration by taking Eq.(24) as for Fig.4a. With this fact the gluon exchanged between the two gluon lines is a Glauber gluon. This indicates that these terms may be factorized with $T_F(x, x)$ according to the experience from Fig.4. The above discussed also applies for the remaining diagrams in Fig.6. In Eq.(29) the terms proportional to $\delta(1-y)$ except the terms containing log's can be identified as hard-pole contributions at one-loop.

The sum of the diagrams in Fig.5 and Fig.6 is then

$$\begin{aligned}
W_T^{(1)} \Big|_{Fig.5+Fig.6} &= -\frac{g_s \alpha_s^2}{(4\pi)^2} \frac{x_0 \sqrt{2x_0}}{(q_\perp^2)^2} \left(-\frac{2}{\epsilon_c} \right) N_c(N_c^2 - 1) \left\{ \delta(x_0 - x) \left[\frac{1+y^2}{(1-y)_+} \left(1 + \frac{y-1}{N_c^2} \right) \right. \right. \\
&\quad \left. \left. - 2\delta(1-y) \ln \frac{q_\perp^2}{Q^2} \right] + \frac{x\delta(1-y)}{x_0(x_0 - x)_+} \right\}.
\end{aligned} \tag{30}$$

With $T_F(x, x)$ in Eq.(14) it is clearly that the terms in $[\dots]$ reproduce the term \mathcal{A}_{sq} in Eq.(8). The last term in the above is not a soft-gluon-pole contribution and will become relevant if one studies the perturbative coefficient functions at the next-to-leading order.

Before ending this section an interesting observation can be made. We observe that SSA calculated here at one-loop is generated through exchange of Glauber gluon at one-loop and it is divergent. This is in contrast to factorizations only with twist-2 operators for Drell-Yan processes. These factorizations are for those differential cross-sections which do not contain T -odd effects. In proving these factorizations, the existence of Glauber gluons brings up the most difficult obstacle[36, 37, 38]. But it is able to show that the divergences caused by Glauber gluons are canceled in differential cross-sections[36, 37, 38]. For the factorization studied here, such divergences are not canceled and they need to be factorized into the twist-3 matrix element with $x_1 = x_2$. This will have some implications for the study of factorizations in the framework of soft collinear effective theories of QCD[39].

5. Additional Contributions

In the previous sections we have used the multi-parton state in Eq.(9) to replace the polarized hadron and determined the factorization form of $W_T^{(1)}$. After the replacement, we have studied SSA essentially in the partonic forward scattering process $\bar{q} + (q + G) \rightarrow \gamma^* + X \rightarrow \bar{q} + q$ or the reversed scattering, where the helicity difference between the initial qg -state and the final q -state is ± 1 . For a real hadron scattering, it is possible that instead of the above scattering one has the forward scattering $\bar{q} + (q + \bar{q}) \rightarrow \gamma^* + X \rightarrow \bar{q} + g$ or the reversed, where the final gluon and initial $q\bar{q}$ come from the polarized hadron. If the total helicity of the $q\bar{q}$ state is zero, this forward scattering will also delivery an additional contribution to SSA besides those given in Eq.(8), because the helicity difference in the scattering is also ± 1 . This has been realized in the study of the evolution of the twist-3 matrix element $T_F(x, x)$ [17].

The additional contribution can be factorized with the twist-3 matrix elements. The factorization can be studied with our multi-parton state in Eq.(9) by adding a $qq\bar{q}$ state as a component:

$$\begin{aligned} |n[\lambda]\rangle &= |q(p, \lambda_q)[\lambda]\rangle + c_1 |q(p_1, \lambda_q)g(k, \lambda_g)[\lambda]\rangle + c_2 |q(k_1, \lambda_1)q(k_2, \lambda_2)\bar{q}(k_3, \lambda_3)[\lambda]\rangle, \\ k_i^\mu &= z_i p, \text{ for } i = 1, 2, 3. \end{aligned} \quad (31)$$

Replacing the transversely polarized hadron h_A , one obtains SSA from the interference of the state $qq\bar{q}$ -state with the qg -state. The interference with the single quark state will not contribute to SSA because of the helicity conservation. By taking one quark from the $qq\bar{q}$ -state and qg -state as a spectator quark, one can have the mentioned forward scattering $\bar{q} + (q + \bar{q}) \rightarrow \gamma^* + X \rightarrow \bar{q} + g$ or the reversed scattering. Therefore, we need effectively to replace, e.g., in the twist-3 matrix elements the state $|h_A\rangle$ with a $q\bar{q}$ state and the state $\langle h_A|$ with a gluon, or in a reversal way. The total helicity state of the $q\bar{q}$ state should be zero. The $q\bar{q}$ state is in color octet in correspondence with the gluon. Taking the quark with k_2 as the spectator, one can simply work out those twist-3 matrix elements at tree-level:

$$\begin{aligned} T_{F,c_2}(x_1, x_2) &= -c_2 \mathcal{N} \pi g_s (N_c^2 - 1) x_0 (x_2 - x_1) \sqrt{2z_1 z_3} \\ &\quad \cdot [\delta(x_1 + z_3)\delta(x_2 - z_1) - \delta(x_2 + z_3)\delta(x_1 - z_1)], \\ T_{\Delta,F,c_2}(x_1, x_2) &= 0. \end{aligned} \quad (32)$$

The factor \mathcal{N} is a normalization factor of the spectator quark state with other trivial factor. The same factor will also appears later in $W_T^{(1)}$ and $T_F(x, x)$. All relevant quantities calculated with the state

Eq.(31) are the sum of contributions studied in previous sections and those studied here. Because of this our study of these contributions can be done separately. It should be noted that there are two possibilities to have one quark as a spectator for the interference with the state in Eq.(31). For simplicity we only present the results of the above possibility in this section, because this will not affect the factorization, i.e., the perturbative coefficient functions. We denote quantities calculated in this way with an index c_2 . We have performed detailed calculations for these two possibilities and obtained the same results for the factorization form of $W_T^{(1)}$ and for the evolution of $T_F(x, x)$ which will be discussed later.

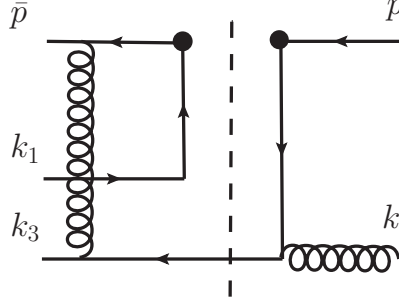


Figure 7: The tree-level diagram contributing to SSA.

Similarly, when we do the replacement of h_A for the hadronic tensor as done for the twist-3 matrix elements in the above, one can obtain nonzero SSA. At tree-level, in the limit $q_\perp \ll Q$, only one diagram gives nonzero contribution. Calculating the contribution in the similar way as shown in [15] and previous sections, we obtain the structure function:

$$W_{T,c_2}^{(1)} = c_2 \mathcal{N} \frac{g_s \alpha_s (N_c^2 - 1)}{4\pi N_c^2 (q_\perp^2)^2} x_0 \sqrt{2z_1 z_3} \delta(1-y) \delta(x-z_1) \frac{x-z_3}{x+z_3}. \quad (33)$$

Comparing with the above T_F we can obtain a factorized form as:

$$W_{T,c_2}^{(1)}(x, y, q_\perp) = \frac{\alpha_s}{(2\pi q_\perp^2)^2 N_c^2} \int_x^1 \frac{dy_1}{y_1} \int_y^1 \frac{dy_2}{y_2} \bar{q}(y_2) T_F(x, x-y_1) \delta(1-\xi_2) (1-2\xi_1), \quad (34)$$

it should be noted that the contribution of $T_F(x_1, x_2)$ calculated in Sect. 3 will not be involved here because this contribution is zero for $x_2 < 0$. It is clear that this part should be added to the factorized form in Eq.(8), i.e., the term \mathcal{A}_h should be modified as:

$$\mathcal{A}_h(x, y_1) = \delta(1-\xi_2) \left[T_F(x, y_1) \frac{1+\xi_1}{(1-\xi_1)_+} + T_{\Delta,F}(x, y_1) + \frac{1-2\xi_1}{N_c^2} T_F(x, x-y_1) \right]. \quad (35)$$

In the above we note that one argument of $T_F(x_1, x_2)$ with $x_2 = x - y_1$ is negative, representing the fact that there is an antiquark from the polarized hadron.

In [16, 17] the evolution of the twist-3 matrix elements have been studied. The results are different. The evolution of the non-singlet part of $T_F(x, x)$ with $x > 0$ is given with $z = x/\xi$ in [17] as:

$$\begin{aligned} \frac{\partial T_F(x, x, \mu)}{\partial \ln \mu} &= \frac{\alpha_s}{\pi} \left\{ \int_x^1 \frac{d\xi}{\xi} \left[P_{qq}(z) T_F(\xi, \xi) + \frac{N_c}{2} \left(\frac{(1+z)T_F(x, \xi) - (1+z^2)T_F(\xi, \xi)}{1-z} + T_{\Delta,F}(x, \xi) \right) \right] \right. \\ &\quad \left. - N_c T(x, x) + \frac{1}{2N_c} \int_x^1 \frac{d\xi}{\xi} [(1-2z)T_F(x, x-\xi) - T_{\Delta,F}(x, x-\xi)] \right\}. \end{aligned} \quad (36)$$

The discrepancies is the following: From [16] the evolution has only the terms in first line. The last term in the first line has a different sign than that of [17]. With our multi-parton state one can calculate $T_F(x_1, x_2)$ to check the evolution. For this we calculate $T_F(x, x)$ with the contribution from the interference with the $qq\bar{q}$ state in the same way showing in the above.

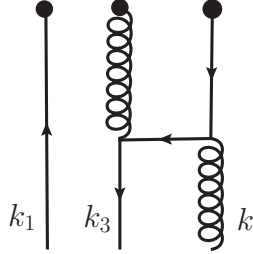


Figure 8: The one-loop diagram to $T_{F,c2}(x, x)$.

For $x > 0$ there is only one diagram at one-loop showing in Fig.8. We obtain the following:

$$T_{F,c2}(x, x, \mu) = c_2 \mathcal{N} \frac{g_s \alpha_s (N_c^2 - 1)}{4N_c} x_0 \sqrt{2y_1 y_3} \delta(x - y_1) \frac{x - y_3}{x + y_3} \left[- \left(\frac{2}{\epsilon_c} - \gamma + \ln 4\pi \right) + \ln \frac{\mu^2}{\mu_c^2} \right]. \quad (37)$$

Adding the results from Sect.3. we have:

$$\begin{aligned} \frac{\partial T_F(x, x, \mu)}{\partial \ln \mu} &= \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left\{ N_c \left[\frac{1+z}{1-z} T_F(x, \xi) + T_{\Delta, F}(x, \xi) \right] + \frac{1}{N_c} (1 - 2z) T_F(x, x - \xi) \right\} \\ &+ \dots \end{aligned} \quad (38)$$

Comparing the above result with that in Eq.(36), we find an agreement except the terms with $T_F(x, x)$ and the last term in Eq.(36). These terms are denoted with \dots in Eq.(38). They can not be obtained at one-loop with our results here. To verify them one has to study them at two-loop level. With our one-loop results at least a part of the discrepancy is solved.

From the interference with the $qq\bar{q}$ state one also expects that there are soft-gluon-pole contributions in Eq.(34). These contributions are generated in a similar way as those studied in Sect. 4. $W_{T,c2}^{(1)}$ receives contributions at one-loop level from similar diagrams in Sect.4., where one replaces the incoming gluon line going through the cut with an antiquark line and the outgoing quark with an outgoing gluon line. Comparing the topology of these diagrams with that of Fig.8 for $T_{F,c2}(x, x)$, one can expect that the contributions to $W_{T,c2}^{(1)}$ from those diagrams can be factorized as the soft-gluon-pole contributions in Eq.(8). We have checked these contributions. The results confirm the above expectation.

6. Summary and Outlook

Because of the helicity-conservation of QCD, SSA of a single quark state can not be generated in the involving forward scattering. We have used multi-parton states in order to have a nonzero SSA in the relevant partonic processes. Using the multi-parton states one can calculate SSA of Drell-Yan processes and relevant twist-3 matrix elements. With these partonic results we can examine and derive the collinear factorization. The collinear factorization for SSA has been derived in the literature based on a diagram

expansion of the relevant hadronic tensor. By using partonic results at tree-level, only the hard-pole contributions can be obtained. The soft-gluon-pole contributions can not be obtained. The reason for this has been discussed.

In this work we have performed the study with multi-parton state at one-loop level in order to examine and identify these soft-gluon-pole contributions in Drell-Yan processes. If we assume that the factorization is correct and there are only hard-pole contributions at leading order of perturbative coefficient functions, we find that a class of one-loop contributions to SSA, which are collinearly divergent, can not be factorized at one-loop order. To correctly factorize collinear divergences appearing in these contributions, one has to add extra terms in the factorization derived with tree-level partonic results. Interestingly these extra terms are just those soft-gluon-pole contributions. Therefore, with our multi-parton states we can re-derive the soft-gluon-pole contributions which have been derived with the diagram expansion in [10].

It is interesting to note that the hard-pole- and soft-gluon-pole contributions in SSA, i.e., in the structure function $W_T^{(1)}$ are at different order of α_s . But, the perturbative coefficient functions are at the same order. The perturbative coefficient functions associated with the soft-gluon-pole contributions, although derived from SSA at one-loop, are at the same order of those associated with the hard-pole contributions, which are derived from SSA at tree-level. This is in contrast with factorizations for differential cross-sections, where only leading twist-2 operators are involved. In these twist-2 factorizations, perturbative coefficient functions at leading order are completely determined by differential cross-sections at tree-level and tree-level matrix elements of involved twist-2 operators. Therefore, our study here also shows an unusual feature of factorizations involving twist-3 operators.

By taking multi-parton state we also find a new contribution to $W_T^{(1)}$. The new contribution comes from the parton process where a $q\bar{q}$ pair with the total helicity $\lambda = 0$ is transmitted into a gluon. This new contribution can be factorized with the twist-3 matrix element. One important twist-3 matrix element is calculated at one-loop in this work. From the result one can derive the evolution of the matrix element. In this work we can solve a part of discrepancy between evolutions derived in [16, 17]. To solve the remaining parts one has to calculate the twist-3 matrix elements with the multi-parton state at two-loop.

In this work we have restrict ourself to the certain relevant partonic processes with one antiquark from the unpolarized hadron. Having succeeded to reproduce the soft-pole-gluon pole contributions with multi-parton states, one can start to analyze with the method presented in this work other relevant partonic processes. E.g., the processes involving one gluon from the unpolarized hadron. In such processes, it is possible to have soft-quark-pole contributions represented by $T_F(0, x)$. One can also use our method to analyze another type of SSA appearing in the case when the momentum of each lepton is measured. In this case the factorization of SSA is so far still different derived from different works [40]. In principle, our method is not restricted to Drell-Yan processes. It can be used in any case where SSA appears. We believe that it has advantages to use our method for analyzing factorizations of SSA and for calculating higher order corrections, because the involved calculations are of standard scattering amplitudes.

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